GENIUS project, neutrino oscillations and Cosmology: neutrinos reveal their nature?* **

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The neutrinoless double beta decay as well as any other laboratory experiment has not been able to answer the question of the neutrino's nature. Hints on the answer are available when neutrino oscillations and $(\beta\beta)_{0\nu}$ are considered simultaneously. In this case phenomenologically interesting neutrino mass schemes can lead to non-vanishing and large values of $\langle m_\nu \rangle$. As a consequence, some schemes with Majorana neutrinos can be ruled out even now. If we assume that in addition neutrinos contribute to Hot Dark Matter then the window for Majorana neutrinos is even more restricted, e.g. GENIUS experiment will be sensitive to scenarios with three Majorana neutrinos.

1. Introduction

There are two main problems in neutrino physics. First is the problem of neutrino mass. In the light of present observations [1] this question seems to be solved, neutrinos are massive particles. The second problem is the one of the neutrino's nature. Massive neutrinos can be either Dirac or Majorana particles. As their visible interactions are left-handed and known

sources generate ultrarelativistic states, it is very difficult to distinguish experimentally between the two.

Dirac spin 1/2 fermions were introduced to describe interactions which are invariant under spatial reflections. Majorana fermions were invented later without special purpose. In those times, parity was conserved and it was generally believed that Majorana particle interactions must be asymmetric under spatial reflection. Today we know that particle characters are not responsible for parity symmetry breaking [2]. Only one property - the charge - discriminates Dirac from Majorana massive fermions. Dirac particles carry charge. Massive Majorana particles must be chargeless and cannot carry static electric or magnetic moments.

Neutrinos are "special" fermions, they have no electric charge and only one "charge" - the lepton number can characterize them. From all present terrestrial experiments it follows that family lepton numbers L_e, L_μ and L_τ are separately conserved and as a result, their sum, the total lepton number $L = L_e + L_\mu + L_\tau$ has the same property.

For massive Dirac neutrinos, flavor lepton numbers can be broken and only L must be conserved. For Majorana neutrinos both, family and total lepton numbers are broken. It is even impossible to define these numbers in the way known from Dirac particles.

We have to stress that Majorana neutrinos are more fundamental objects and naturally arise in most extensions of the Standard Model. Only in models where the lepton number (L and $B - L$) is conserved, neutrinos are Dirac particles. However, there are many arguments to abandon lepton number conservation. It is not a fundamental quantity, unlike electric charge and does not govern the dynamics. Also, lepton number violation is naturally induced by the presence of right-handed neutrinos (ν_R) which are usually necessary to form the Dirac mass term ($\overline{\nu}_L \nu_R$). In spite of these obvious theoretical arguments, supporting the Majorana nature of neutrinos, finding direct experimental indications which would determine the neutrino character is very important.

It is common belief that the first place where the nature of massive neutrinos will be revealed is the neutrinoless double β decay of nuclei, $(\beta\beta)_{0\nu}$. Many experimental searches for $(\beta\beta)_{0\nu}$ decay of different nuclei have been done and are presently underway [3]. Unfortunately, up to now this decay has not been found and the experimental data can only help to estimate the lower bound on the life times of $(\beta\beta)_{0\nu}$ decay modes. The most stringent limit was found in the germanium Heidelberg-Moscow experiments. Their latest result on the half-life time is [4]

$$T_{1/2}^{o\nu}(Ge) > 5.7 \times 10^{25} \text{ year (at 90\% CL)} \tag{1}$$

from which the following upper bound on the effective Majorana mass was

found

$$| \langle m_\nu \rangle | = \left| \sum_i U_{ei}^2 m_i \right| < 0.2 \ eV. \tag{2}$$

The above number has been used to restrict many aspects of the neutrino mass spectrum or the solar neutrino mechanism [5]. We propose an opposite way of thinking. Using the present data from oscillation experiments, and tritium β decay we can find the modules $|U_{ei}|$ of mixing matrix elements and the possible values of neutrino masses m_i. Then we check whether the bound Eq. (2) is satisfied or not. If it is, the problem is unsolved. If, however the bound Eq. (2) is not satisfied, then neutrinos are Dirac particles. In the latter case the effective mass is calculated as [6]

$$\sum_{l=1}^{n} U_{el}^2 m_l \Rightarrow \sum_{l=1}^{n} \frac{1}{2} \left[(-iU_{el})^2 + (U_{el})^2 \right] m_l = 0 \tag{3}$$

and is strictly equal zero.

Ambitious plans [7] are to shift up the limit Eq. (1) and to move the upper limit of $< m_\nu >$ down to 0.02 eV (using a tank of 1 ton of Germanium, after one year) or in a further time scale even to 0.006 eV (1t, 10 years).

In a previous work [8] we have considered three neutrino mixing schemes. Here we present analytical results for both three and four neutrino mixing scenarios. Information about the same subject with numerical estimations is given in [9]. In the next Chapter we summarize the efforts undertaken in order to find lepton number violating processes. Explanation is given of why the family and total lepton numbers are so strongly conserved. In Chapter 3 we collect all the relevant information about mixing matrix elements and masses extracted from experimental data. Four presently accepted neutrino mass schemes which cover the case of three and four neutrino mixing are discussed. All necessary information from oscillation experiments, tritium β decay and cosmology is given. Chapter 4 is the main of the paper. All data are connected together with the bound on effective neutrino mass from $(\beta\beta)_{0\nu}$, and restrictions on various neutrino mass schemes are presented. Conclusions are to be found in Chapter 5.

2. Lepton numbers and neutrino character of light SM neutrino states

In order to explain the lack of lepton flavor violating processes, the concept of the flavor lepton number L_α [10] followed by the idea of the total lepton number L [11] have been introduced. The upper bounds on

branching ratios of L_α violating processes are very small, for instance:

$$BR(\mu^- \to e^-\gamma) < 4.9 \cdot 10^{-11}, \qquad BR(\mu^- \to e^+e^-e^-) < 1.0 \cdot 10^{-12},$$

$$BR(\pi^0 \to \mu^-e^+) < 1.72 \cdot 10^{-8}, \quad BR(K_L^0 \to \mu^-e^+) < 3.3 \cdot 10^{-11}, \qquad (4)$$

$$BR(\tau^- \to \mu^-\gamma) < 4.2 \cdot 10^{-6}.$$

In the frame of the SM with massless neutrinos the above processes are strictly forbidden. If neutrinos are massive, then in analogy to the quark sector neutrinos should mix and lepton numbers are not conserved. However, these effects must be very small, below sensitivity of processes given in Eq. (4). That means that the concept of leptons numbers L, L_α is still usefull, at least in all neutrino nonoscillation phenomena. For Dirac neutrinos represented by a bispinor Ψ_D it is possible to change the phase of the field

$$\Psi_D \to e^{i\alpha}\Psi_D. \qquad (5)$$

The charge connected with such a global gauge transformation is just the flavor charge operator. This operator can, but not necessarily must, commute with the interaction Hamiltonian, $[L_\alpha, H] = 0$ for a massless, $[L_\alpha, H] \neq 0$ for a massive neutrino. Majorana neutrinos on the other hand are described by self-conjugate fields

$$\Psi_M = \Psi_M^c \equiv C\bar{\Psi}_M^T, \qquad (6)$$

and it is not possible to define the same kind of gauge transformation as in Eq. (5). There is then no special reason why L_α and L should be conserved for Majorana neutrinos. All processes in Eq. (4) break L_α but not L, so they can be realized by both kind of neutrinos at the one loop level. At this level only very heavy, nonstandard, neutrinos matter [12]. We do not go to details and focus only on direct effects connected with light, SM neutrinos. Let us mention only that in see-saw models heavy neutrino effects are also negligible, both at tree [13] and loop levels [14]. To make life easier and to understand how processes with Majorana (Dirac) neutrinos mimic family L_α and total lepton L numbers conservation let us consider a tree level process of electron (positron) production using electron and muon neutrinos scattering on nuclear target

$$\nu_{e(\mu)}N \to e^{\pm}X. \qquad (7)$$

Let us define the connection between flavor ν_α and massive ν_i states in the following way

$$|\nu_\alpha(\lambda = -\frac{1}{2})\rangle = \sum_i U_{\alpha i}|\nu_i(\lambda = -\frac{1}{2})\rangle \qquad (8)$$

for negative helicity states and

$$|\nu_\alpha(\lambda = +\frac{1}{2})\rangle = \sum_i U_{\alpha i}^*|\nu_i(\lambda = +\frac{1}{2})\rangle \qquad (9)$$

for $\lambda = +\frac{1}{2}$. In the same way Weyl particle and antiparticle states are connected. Note that Eqs. (8-9) for massive particles seems to be in contradiction with the special theory of relativity. The real problem is that states on the right hand side of Eqs. (8-9) can not be defined, in general [15]. However, the left-handed interaction cannot change the neutrino helicity and it is practically impossible to find a real frame moving faster than the neutrino itself (neutrinos are ultrarelativistic) and the relations Eqs. (8-9) can be safely used [15]. This is what is usually considered to be true when neutrino oscillation phenomena are discussed. To be more general, let us assume that there is also a right-handed neutrino interaction [1]

$$L_{CC} = \frac{g}{\sqrt{2}}\left[A_L\left(\overline{N}_i\gamma^\mu P_L(U^T)_{i\alpha}l_\alpha\right)W_{L\mu}^+ + A_R\left(\overline{N}_i\gamma^\mu P_R(U^\dagger)_{i\alpha}l_\alpha\right)\right]W_{R\mu}^+ + h.c. \qquad (10)$$

Then in the ultrarelativistic regime ($m_i << E$) using the unitarity of the U matrix, the following amplitudes to the $\mathcal{O}\left(\left(\frac{m_i}{E}\right)^2\right)$ order are obtained:

$$A\left(\nu_e(-1/2) \rightarrow e^-\right) = A(e^-)\left[A_L^* + A_R^*\sum_i \frac{m_i}{2E}\left(U_{ei}\right)^2\right], \qquad (11)$$

$$A\left(\nu_e(+1/2) \rightarrow e^-\right) = A(e^-)\left[A_L^*\sum_i \frac{m_i}{2E}\left(U_{ei}^*\right)^2 + A_R^*\right], \qquad (12)$$

$$A\left(\nu_\mu(-1/2) \rightarrow e^-\right) = A(e^-)\left[-A_L^*\sum_i \frac{m_i^2}{8E^2}U_{\mu i}U_{ei}^* + A_R^*\sum_i \frac{m_i}{2E}U_{\mu i}U_{ei}\right], \qquad (13)$$

$$A\left(\nu_\mu(+1/2) \rightarrow e^-\right) = A(e^-)\left[A_L^*\sum_i \frac{m_i}{2E}U_{\mu i}^*U_{ei}^* - A_R^*\sum_i \frac{m_i^2}{8E^2}U_{\mu i}^*U_{ei}\right] \qquad (14)$$

and

$$A\left(\nu_e(-1/2) \rightarrow e^+\right) = A(e^+)\left[-A_L\sum_i \frac{m_i}{2E}\left(U_{ei}\right)^2 + A_R\right], \qquad (15)$$

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[1] For Dirac neutrinos there are actually two independent neutrino mixing matrices in left- and right- handed charged currents [16]. Even for Majorana neutrinos these could be in principle different (as is e.g. the case of see-saw type models where the light neutrino mixing matrix in the right-handed current is dumped by the heavy neutrino mass scale). These simplifications do not spoil the general idea given here.

$$A\left(\nu_e(+1/2) \to e^+\right) = A(e^+)\left[-A_L + A_R \sum_i \frac{m_i}{2E}\left(U_{ei}^*\right)^2\right], \tag{16}$$

$$A\left(\nu_\mu(-1/2) \to e^+\right) = A(e^+)\left[-A_L \sum_i \frac{m_i}{2E}U_{\mu i}U_{ei} + A_R \sum_i \frac{m_i^2}{8E^2}U_{\mu i}U_{ei}^*\right], \tag{17}$$

$$A\left(\nu_\mu(+1/2) \to e^+\right) = A(e^+)\left[A_L \sum_i \frac{m_i^2}{8E^2}U_{\mu i}^*U_{ei} + A_R^* \sum_i \frac{m_i}{2E}U_{\mu i}^*U_{ei}^*\right] \tag{18}$$

where $A(e^+)$ and $A(e^-)$ are appropriate amplitudes for massless neutrinos. In the approximation $\frac{m_i}{E} << 1$ and $|A_L| >> |A_R|$ only two cross sections for electron production by a $\nu_e(\lambda = -1/2)$ beam Eq. (11) and positron production by a $\nu_e(\lambda = +1/2)$ beam Eq. (16) are large enough to be seen

$$\sigma\left(\nu_e(-1/2) \to e^-\right) \sim |A(e^-)|^2, \tag{19}$$

$$\sigma\left(\nu_e(+1/2) \to e^+\right) \sim |A(e^+)|^2. \tag{20}$$

All other helicity cross sections are suppressed by factors

$$\left(\frac{m_i}{E}\right)^2, \quad \frac{m_i}{2E}|A_R| \quad \text{or} \quad |A_R|^2, \tag{21}$$

and for instance, for $m_i \simeq 1$ eV and $E \simeq 1$ MeV we have $\left(\frac{m_i}{E}\right)^2 \simeq 10^{-12}$. Such factors cause that the cross sections for flavor lepton number L_α violating processes Eqs. (13-14), Eqs. (17-18) are invisibly small. The total lepton L non-conserving processes Eqs. (12,15), share the same property. Neglecting the factors from Eq. (21), our amplitudes are identical to those of massless Weyl neutrinos whose family and total lepton numbers are strictly conserved. Turning our results around we can see that processes where neutrino masses (and right-handed currents) are not important give no chance to distinguish Dirac from Majorana neutrinos. Could CP phases help? In the case of Dirac [Majorana] neutrinos the mixing matrix U has $(n - 1)(n - 2)/2\,[n(n - 1)/2]$ phases. Let us look into processes where the neutrino mass is important. Though the transition probability of neutrino oscillations depends on CP phases, the physical phases by which the neutrino mixing matrices differ do not enter into transition probabilities and the results are the same for Dirac and Majorana neutrinos [16, 17]. The neutrino mass distortion measured in tritium β decay is a function of absolute values of mixing matrix elements (see next chapter) so it is not sensible to CP phases, either.

There are also processes which do not conserve the total lepton number in which only Majorana neutrinos could participate. Since many years the

most promising investigation along this line is connected with the neutrino-less double beta decay.

Surprisingly, we will see that even if this process is not observed, it can solve the problem of the nature of neutrinos, when augmented with Cosmology (assuming neutrinos as Hot Dark Matter) and neutrino oscillations results.

3. Neutrino masses and mixing matrix U_{ei} elements

There are two completely different situations which depend on the present status of the LSND experiment. Three light neutrinos are necessary to explain solar [18] and atmospheric [19] anomalies. With the LSND result [20] an extra light neutrino must be introduced.

3.1. Three neutrinos scenario

For neutrino mixing 3 flavor states $(\nu_e, \nu_\mu, \nu_\tau)$ are related to 3 eigenmass states (ν_1, ν_2, ν_3) through[21]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{22}$$

Our concern is about the first row of the mixing matrix. We use the standard parameterization [22]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \tag{23}$$

To explain the solar neutrino anomaly the mass splitting between two neutrinos must be extremely tiny, $\delta m^2_{sun} \simeq 10^{-5} \div 10^{-11}\ eV^2$. Only slightly larger mass splitting between neutrino masses is needed in the case of atmospheric oscillations, $\delta m^2_{atm} \simeq 10^{-2} \div 10^{-3}\ eV^2$. These relations leave us with two possible neutrino mass scenarios: $\delta m^2_{12} = \delta m^2_{sun}$, $\delta m^2_{23} \simeq \delta m^2_{13} = \delta m^2_{atm}$ (Scheme A_3, Fig.1) and $\delta m^2_{23} = \delta m^2_{sun}$, $\delta m^2_{12} \simeq \delta m^2_{13} = \delta m^2_{atm}$ (Scheme B_3, Fig.1) Reactor experiments are of the so-called short baseline and are able to measure the neutrino mass splitting of the order $\delta m^2 < 10^{-3}$. Then we can neglect terms with $\delta m^2_{sun} \ll 10^{-3}$ and the disappearance probability for $\bar{\nu}_e$ reactor neutrino oscillations is (the following discussion is given for the A_3 scheme)

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = P_{\nu_e \to \nu_e} = 1 - \sin^2 2\Theta_{13} \sin^2 \Delta_{reactor}, \tag{24}$$
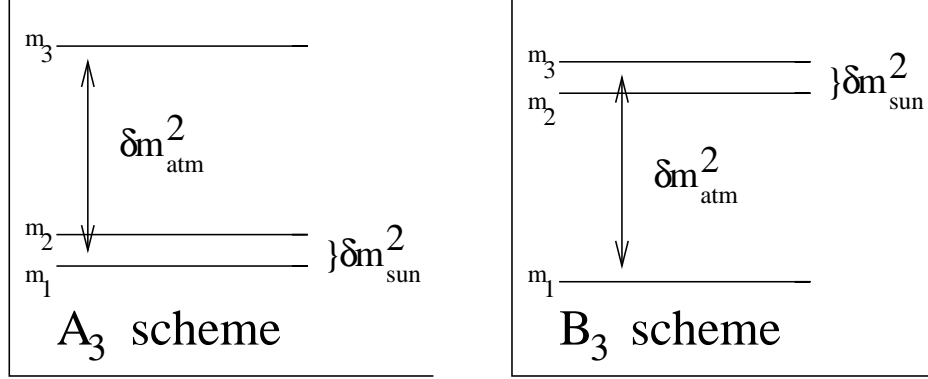
Fig. 1. Two possible neutrino mass spectra which can describe the oscillation data.

where

$$\Delta_{reactor} = \Delta_{23}(L_{reactor}, E_{reactor}), \quad \Delta_{ij}(L, E) = \frac{1.27 \times \delta m_{ij}^2 [eV^2] L[km]}{E[GeV]}.$$

In reactor experiments the disappearance of $\bar{\nu}_e$ is not seen, which means that $\sin^2 2\Theta_{13}$ must be small. CHOOZ gives [23]

$$\sin^2 2\Theta_{13} < 0.18, \tag{25}$$

and two solutions for Θ_{13} can be found:

$$\sin^2 \Theta_{13} < 0.05 \quad \text{or} \quad \sin^2 \Theta_{13} > 0.95. \tag{26}$$

The observed ν_μ neutrino deficit from the atmosphere is favorable describe by a $\nu_\mu \to \nu_\tau$ transition where matter effects are not important ($\Delta_{atm} = \Delta_{23}(L_{atm}, E_{atm})$):

$$P_{\nu_\mu \to \nu_\tau} = \sin^2 2\Theta_{23} \cos^4 \Theta_{13} \sin^2 \Delta_{atm}. \tag{27}$$

We know that the atmospheric neutrino mixing is very large [24]

$$0.72 \leq \sin^2 2\Theta_{23} \cos^4 \Theta_{13} \leq 1 \quad \text{and} \quad \delta m_{atm}^2 \simeq 4 \times 10^{-3} \ eV^2 \tag{28}$$

and only a small value of $\sin^2 \Theta_{13}$ in Eq. (26) is compatible with the bound in Eq. (28). The recent fit to the new (830-920 days) atmospheric data of Superkamiokande gives the minimum of χ^2 for [25]

$$\sin^2 \Theta_{13} = 0.03. \tag{29}$$

Similar values ($\sin^2 \Theta_{13} < 0.03 \div 0.04$ [23, 26]) are given by the reactor data.

In all solar neutrino experiments the deficit of electron antineutrinos is measured and four different solutions are possible [27]. The first, "just so" solution, is based on the hypothesis of neutrino oscillations in vacuum (VO), $\delta m^2_{sun} \sim 10^{-10} \ eV^2$ in this case. The other three solutions are based on the Wolfenstein[28]-Mikheyev-Smirnov [29] mechanism of coherent neutrino scattering in matter (so called small mixing angle (SMA MSW), large mixing angle (LMA MSW) and low δm^2 (LOW MSW) solutions).

For VO the ν_e disappearance probability is given by ($\Delta_{sun} = \Delta_{12}(L_{sun}, E_{sun})$)

$$P^{sun}_{\nu_e \to \nu_e} \ = \ 1 - \frac{1}{2}\sin^2 2\Theta_{13} - \sin^2 2\Theta_{12} \cos^4 \Theta_{13} \sin^2 \Delta_{sun}. \qquad (30)$$

This expression can be rewritten in the form

$$P^{sun}_{\nu_e \to \nu_e} = \cos^4 \Theta_{13} \left(1 - \sin^2 2\Theta_{12} \sin^2 \Delta_{sun} \right) + \sin^4 \Theta_{13}. \qquad (31)$$

Taking into account that $\sin^4 \Theta_{13} \simeq 0$ (Eq. (29)) we get

$$P^{sun}_{\nu_e \to \nu_e} \simeq 1 - \sin^2 2\Theta_{sun} \sin^2 \Delta_{sun}. \qquad (32)$$

where $\Theta_{sun} \simeq \Theta_{12}$.

Similarly we get for the case of the MSW solution:

$$P^{sun(MSW)}_{\nu_e \to \nu_e} \simeq 1 - \sin^2 2\tilde{\Theta}_{sun} \sin^2 \tilde{\Delta}_{sun}. \qquad (33)$$

where now

$$\sin^2 2\tilde{\Theta}_{sun} = \frac{\sin^2 2\Theta_{sun}}{\left[\left(\frac{A}{\delta m^2_{sun}} - \cos 2\Theta_{sun} \right)^2 + \sin^2 2\Theta_{sun} \right]^{1/2}}. \qquad (34)$$

$\tilde{\Delta}_{sun}$ includes the effective neutrino mass parameter with δm^2_{sun} replaced by

$$\tilde{\delta} m^2_{sun} = \delta m^2_{sun} \left[\left(\frac{A}{\delta m^2_{sun}} - \cos 2\Theta_{sun} \right)^2 + \sin^2 2\Theta_{sun} \right]^{1/2}, \qquad (35)$$

where $A = 2\sqrt{2} G_F E N_e$ (N_e - electron number density).

From Eq. (35) we can see that in order to fulfill the resonance condition we need for $\delta m^2_{sun} > 0$ [30]

$$\cos 2\Theta_{sun} > 0, \qquad (36)$$

and this means that

$$\cos \Theta_{12} > \sin \Theta_{12}. \qquad (37)$$

Possible solutions	$\sin^2 2\Theta_{sun}$ [95 % c.l.]	Best fits	
		$\sin^2 2\Theta_{sun}$	δm^2
MSW SMA	0.001 - 0.01	0.0065	$5.2 \times 10^{-6} eV^2$
MSW LMA	0.59 - 0.98	0.77	$2.94 \times 10^{-5} eV^2$
MSW LOW	0.68-0.98	0.9	$1.24 \times 10^{-7} eV^2$
VO		0.93	$4.42 \times 10^{-10} eV^2$

Table 1. The allowed ranges and best fit values of $\sin^2 2\Theta_{sun}$ and δm^2 for different types of solar neutrino oscillations.

Many fits have been done to the solar neutrino data [31]. The results of the fit [32] which takes into account the full set of measurements (rates, energy spectrum, day-night asymmetry in the case of the MSW solution and seasonal variation for VO solution) are presented in Table I. For VO only the best fit value $\sin^2 2\Theta_{sun}$ is given in [33].

For a scheme B_3 a change $U_{e3} \leftrightarrow U_{e1}$ must be done.

3.2. Four neutrinos scenario

The electron (anti)neutrino appearance in the LSND experiment [20, 34] can be explained by $\nu_\mu \to \nu_e$ oscillation with additional large δm^2 scale

$$\delta m^2_{LSND} \sim 0.2 \div 2 \text{ eV}^2. \tag{38}$$

In principle there are six possible four-neutrino mass schemes with three different scales of δm^2. They are widely discussed in literature [35] and it is known that only two schemes (Fig.2) are accepted by reactor, LSND, solar and atmospheric neutrino data.

As the parameterization of the 4×4 neutrino mixing matrix is very complicated [36] in the case when all entries of the mass matrix are nonzero we will use only the symbolic denotations and take

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}. \tag{39}$$

For the short baseline experiment (and for scheme A_4) the probability of disappearance of ν_e neutrinos is given by ($\Delta_{SBL} = \Delta_{32}(L_{SBL}, E_{SBL})$)

$$P_{\nu_e \to \nu_e} = 1 - c_e(1 - c_e) \sin^2 \Delta_{SBL} \tag{40}$$
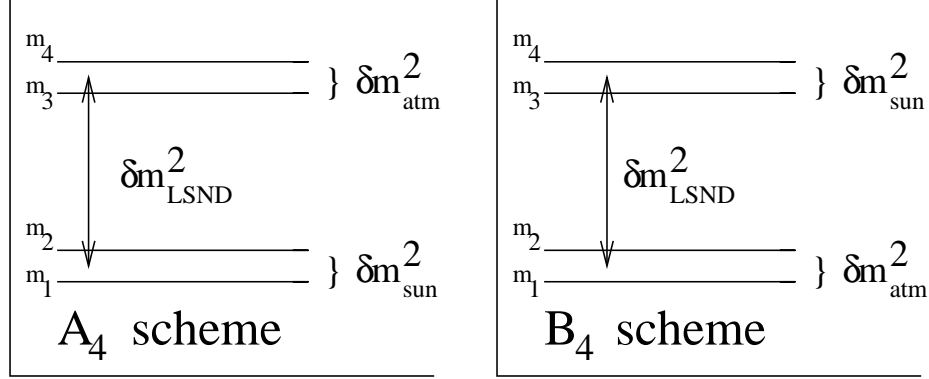
where

$$c_e = |U_{e3}|^2 + |U_{e4}|^2.$$

Fig. 2. Two accepted by present data four-neutrino schemes. In the scheme A_4 $\delta m_{sun}^2 = \delta m_{12}^2$, $\delta m_{atm}^2 = \delta m_{34}^2$ with opposite situation in B_4 scheme where $\delta m_{sun}^2 = \delta m_{34}^2$, $\delta m_{atm}^2 = \delta m_{12}^2$. In both schemes $\delta m_{LSND}^2 \simeq \delta m_{13}^2 \simeq \delta m_{14}^2 \simeq \delta m_{23}^2 \simeq \delta m_{24}^2$.

Again we can implement the CHOOZ result [23] and get

$$4c_e(1 - c_e) < 0.18, \tag{41}$$

and there are two solutions for c_e, namely

$$c_e < 0.05 \ \text{ or } \ c_e > 0.95. \tag{42}$$

On the other hand the deficit of solar neutrinos in the VO scenario and in four-neutrino language reads ($\Delta_{sun} = \Delta_{12}(L_{sun}, E_{sun})$)

$$P_{\nu_e \to \nu_e}^{(4)} = \left(1 - |U_{e3}|^2 - |U_{e4}|^2\right)^2 \left[1 - \frac{4|U_{e1}|^2|U_{e2}|^2}{(|U_{e1}|^2 + |U_{e2}|^2)^2} \sin^2 \Delta_{sun}\right] + |U_{e3}|^4 + |U_{e4}|^4. \tag{43}$$

By comparison with the two flavour oscillations formula

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\Theta_{sun} \sin^2 \Delta_{sun} \tag{44}$$

we can see that the factor $\left(1 - |U_{e3}|^2 - |U_{e4}|^2\right)^2$ must be close to one, so only one solution of Eq. (42) is possible, namely $c_e < 0.05$. For small values of c_e the probability $P_{\nu_e \to \nu_e}^{(4)}$ is well described in the frame of the 3 neutrino scenario by $P_{\nu_e \to \nu_e}^{(3)}$ (Eq. (31)) with the following substitutions

$$|U_{e3}|^2 + |U_{e4}|^2 \ \to \ \sin^2 \Theta_{13} < 0.05,$$

$$\frac{|U_{e1}|^2}{|U_{e1}|^2 - |U_{e2}|^2} \ = \ \cos^2 \Theta_{12}, \quad \frac{|U_{e2}|^2}{|U_{e1}|^2 - |U_{e2}|^2} = \sin^2 \Theta_{12}. \tag{45}$$

That means that fitted parameters are the same as in 3-neutrino case

$$\sin^2 \Theta_{13} = |U_{e3}|^2 + |U_{e4}|^2 \equiv c_e < 0.05. \tag{46}$$

The MSW and VO solutions are described by $\sin^2 2\Theta_{12} = \sin^2 2\Theta_{sun}$ with the same values as in 3 neutrino scenario given in Table I. For a scheme B_4 a change $U_{e3(4)} \leftrightarrow U_{e1(2)}$ must be done.

3.3. Tritium beta decay

Other constraints on neutrino masses and mixings come from the observation of the end of the Curie plot for the tritium β decay. Two collaborations from Mainz and Troitsk give similar results for the upper limit (95% of c.l.) on the effective electron neutrino mass

$$\langle m_{\nu_e} \rangle_\beta = \left[\sum_{i=1}^{n} |U_{ei}|^2 m_i^2 \right]^{1/2}, \tag{47}$$

$$\langle m_{\nu_e} \rangle_\beta \;<\; 2.8 \; eV \quad \text{Mainz Collaboration [37]},$$
$$\langle m_{\nu_e} \rangle_\beta \;<\; 2.5 \; eV \quad \text{Troitsk Collaboration [38]}.$$

Both collaborations have ambitious plans to probe the mass region below 1 eV during the next five years [39].

3.4. Cosmological bounds

There are also astrophysical and cosmological bounds on neutrino masses and mixings. All this information depends on many other assumptions (as e.g. nonzero cosmological constant Λ) and is not as strict as laboratory data. We will take into account only one data which comes from the so called dark matter problem. If neutrinos compose all invisible matter in the Universe then [40]

$$\sum m_\nu \leq 30 \; eV. \tag{48}$$

If only 20 % of all dark matter is formed by neutrinos (the so called Hot Dark Matter) then

$$\sum m_\nu \simeq 6 \; eV. \tag{49}$$

The best fit to many cosmological quantities is obtained if around 70 % of dark matter is given by nonzero cosmological constant, 24 % by Cold Dark Matter and 6 % by Hot Dark Matter. In such a case

$$\sum m_\nu \simeq 2 \; eV. \tag{50}$$

4. Neutrinoless double beta decay and constraints on neutrino nature

Neutrinoless double beta decay is sensitive to the first element of the neutrino mass matrix

$$m_{\alpha\beta} = \sum_{i=1}^{n} U_{\alpha i} U_{\beta i} m_i \tag{51}$$

and luckily, is very well constrained. This is not the case of other entries which are also measured in various laboratory experiments, for instance

$$
\begin{aligned}
|m_{e\mu}| & \quad \text{in } Ti + \mu^- \to Ca + e^+ \\
|m_{\mu\mu}| & \quad \text{in } K^+ \to \pi^- \mu^+ \mu^+ \\
|m_{e\tau}|, |m_{\mu\tau}|, |m_{\tau\tau}| & \quad \text{in HERA from } e^- p \to \nu_e l^- l'^- X.
\end{aligned} \tag{52}
$$

All these quantities have quite large bounds, in the MeV-GeV range [41] e.g.

$$
\begin{aligned}
|m_{e\mu}| & < 17 \quad \text{MeV}, \\
|m_{\mu\mu}| & < 500 \quad \text{GeV}, \\
|m_{e\tau}| & < 8.4 \quad \text{TeV}.
\end{aligned} \tag{53}
$$

The mixing matrix for Majorana neutrino has 3(6) phases for 3(4) neutrinos so we have

$$|\langle m_\nu \rangle| = \left| |U_{e1}|^2 m_1 + e^{2i\phi_2}|U_{e2}|^2 m_2 + e^{2i\phi_3}|U_{e3}|^2 m_3 \right|, \tag{54}$$

$$|\langle m_\nu \rangle| = \left| |U_{e1}|^2 m_1 + e^{2i\phi_2}|U_{e2}|^2 m_2 + e^{2i\phi_3}|U_{e3}|^2 m_3 + e^{2i\phi_4}|U_{e4}|^2 m_4 \right|, \tag{55}$$

for $n = 3(4)$ neutrinos, respectively.

We should stress that all our results are obtained in the approximation in which the lightest of neutrinos $(m_\nu)_{min}$ is heavier than the difference of squares of neutrino masses responsible for solar neutrino oscillations $((m_\nu)_{min} >> \delta m^2_{sun})$.

4.1. A schemes

Let us first discuss the schemes A_3 and A_4. We have

- for A_3

$$m_1 = (m_\nu)_{min},$$

$$m_2 = \sqrt{(m_\nu)^2_{min} + \delta m^2_{sun}} \simeq m_1,$$

$$m_3 = \sqrt{(m_\nu)^2_{min} + \delta m^2_{atm} + \delta m^2_{sun}} \simeq \sqrt{(m_\nu)^2_{min} + \delta m^2_{atm}}, \tag{56}$$

- for A_4

$$m_1, m_2 \quad \text{as for } A_3,$$

$$m_3 = \sqrt{(m_\nu)^2_{min} + \delta m^2_{LSND} + \delta m^2_{sun}} \simeq \sqrt{(m_\nu)^2_{min} + \delta m^2_{LSND}},$$

$$m_4 = \sqrt{m_3^2 + \delta m^2_{atm}} \simeq m_3. \tag{57}$$

Using the relation

$$min|z_1 + z_2 + z_3 + z_4| = \begin{cases} |z_3 + z_4|_{min} - |z_1 + z_2|_{max} > 0, \\ 0, \\ |z_1 + z_2|_{min} - |z_3 + z_4|_{max} > 0, \end{cases} \tag{58}$$

we get for both schemes

$$|\langle m_\nu \rangle|_{min} = \begin{cases} s_{min} - \left(|U_{e1}|^2 m_1 + |U_{e2}|^2 m_2\right) & (m_\nu)_{min} \in (0, x_1^A), \\ 0 & (m_\nu)_{min} \in (x_1^A, x_2^A), \\ \left||U_{e1}|^2 m_1 - |U_{e2}|^2 m_2\right| - s_{max} & (m_\nu)_{min} > x_2^A, \end{cases} \tag{59}$$

where

$$s_{\min} = s_{\max} = c_e m_3, \quad (A_3) \text{ scheme} \tag{60}$$

$$s_{\min} = \left||U_{e3}|^2 m_3 - |U_{e4}|^2 m_4\right|, \quad (A_4) \text{ scheme} \tag{61}$$

$$s_{\max} = |U_{e3}|^2 m_3 + |U_{e4}|^2 m_4, \quad (A_4) \text{ scheme}. \tag{62}$$

x_1^A and x_2^A are the values of $(m_\nu)_{min} = m_1 > 0$ for which

$$s_{\min} - \left(|U_{e1}|^2 m_1 + |U_{e2}|^2 m_2\right) = 0 \text{ and } \left||U_{e1}|^2 m_1 - |U_{e2}|^2 m_2\right| - s_{\max} = 0 \tag{63}$$

respectively.

In both schemes there is (in agreement with Eq. (37) we take $|U_{e1}|^2 > |U_{e2}|^2$)

$$|U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 = (m_\nu)_{min} (1 - c_e) \tag{64}$$

and

$$\left| |U_{e1}|^2 \, m_1 - |U_{e2}|^2 \, m_2 \right| = (m_\nu)_{min} \, (1 - c_e) \, \sqrt{1 - \sin^2 2\theta_{sun}}. \quad (65)$$

In the A_4 scheme we do not know $|U_{e3}|^2$ and $|U_{e4}|^2$ separately and only approximate values for $s_{\max}$ can be found

$$s_{\max} = |U_{e3}|^2 \, m_3 + |U_{e4}|^2 \, \sqrt{m_3 + \delta m_{atm}^2} \approx c_e \sqrt{(m_\nu)_{min}^2 + \delta m_{LSND}^2}. \quad (66)$$

The $s_{\min}$ is unknown so the region of $(m_\nu)_{min} \in \left(0, x_1^A\right)$ cannot be checked precisely. We can find however that in both schemes ($\delta m^2 = \delta m_{atm}^2$ or δm_{LSND}^2)

$$|\langle m_\nu \rangle|_{\min} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (67)$$

$$\leq s_{\max} - |U_{e1}|^2 \, m_1 - |U_{e2}|^2 \, m_2 = c_e \sqrt{(m_\nu)_{min}^2 + \delta m^2} - (m_\nu)_{min} \, (1 - c_e)$$

$$< \begin{cases} c_e \sqrt{\delta m_{atm}^2} \approx 0.002 & eV \text{ for } A_3, \\ c_e \sqrt{\delta m_{LSND}^2} \approx 0.03 & eV \text{ for } A_4. \end{cases}$$

The region $(m_\nu)_{min} > x_2^A$ is more interesting. In both schemes this region occurs if

$$(m_\nu)_{min} \, (1 - c_e) \, \sqrt{1 - \sin^2 2\theta_{sun}} - c_e \sqrt{(m_\nu)_{min}^2 + \delta m^2} \geq 0, \quad (68)$$

from which the condition for $\sin^2 2\theta_{sun}$ follows

$$\sin^2 2\theta_{sun} \leq \frac{1 - 2c_e}{(1 - c_e)^2}. \quad (69)$$

For such values of mixing angle θ_{sun} we can find x_2^A

$$x_2^A = \frac{\delta m^2 c_e}{\sqrt{(1 - c_e)^2 \, (1 - \sin^2 2\theta_{sun})} - c_e}. \quad (70)$$

4.2. B schemes

For B_3 and B_4 schemes the neutrino masses are connected with the lightest neutrino mass as follows:

- for B_3

$$\begin{aligned} m_1 &= (m_\nu)_{min}, \\ m_2 &= \sqrt{(m_\nu)_{min}^2 + \delta m_{atm}^2}, \\ m_3 &= \sqrt{(m_\nu)_{min}^2 + \delta m_{atm}^2 + \delta m_{sun}^2} \approx m_3 \end{aligned} \qquad (71)$$

- for B_4

$$m_1, m_2 \text{ as in } B_3,$$
$$m_3 = \sqrt{(m_\nu)^2_{min} + \delta m^2_{atm} + \delta m^2_{LSND}}, \tag{72}$$
$$m_4 = \sqrt{m_3^2 + \delta m_{sun}} \approx m_3.$$

Using the relation Eq. (58) we obtain

$$|\langle m_\nu \rangle|_{\min} = \begin{cases} w_{\min} - \left(|U_{e3}|^2 m_3 + |U_{e4}|^2 m_4 \right) > 0, & (m_\nu)_{min} \in \left(0, x_1^B \right) \\ 0, & (m_\nu)_{min} \in \left(x_1^B, x_2^B \right) \\ \left| |U_{e3}|^2 m_3 - |U_{e4}|^2 m_4 \right| - w_{\max}, & (m_\nu)_{min} > x_2^B. \end{cases} \tag{73}$$

where ($c_e = |U_{e1}|^2$ or $|U_{e1}|^2 + |U_{e2}|^2$)

$$w_{\min} = w_{\max} = c_e m_1, \quad (B_3) \text{ scheme} \tag{74}$$
$$w_{\min} = \left| |U_{e1}|^2 m_1 - |U_{e2}|^2 m_2 \right|, \quad (B_4) \text{ scheme} \tag{75}$$
$$w_{\max} = |U_{e1}|^2 m_1 + |U_{e2}|^2 m_2,, \quad (B_4) \text{ scheme.} \tag{76}$$

x_1^B and x_1^B are solutions of the equations

$$w_{\min} - \left(|U_{e3}|^2 m_3 + |U_{e4}|^2 m_4 \right) = 0, \text{ and } \left| |U_{e3}|^2 m_3 - |U_{e4}|^2 m_4 \right| - w_{\max} = 0, \tag{77}$$

respectively.

Now there is (once more we assume $|U_{e3}| > |U_{e4}|$)

$$|U_{e3}|^2 m_3 + |U_{e4}|^2 m_4 = (1 - c_e) \sqrt{(m_\nu)^2_{min} + \delta m^2}, \text{ and} \tag{78}$$

$$\left| |U_{e3}|^2 m_3 - |U_{e4}|^2 m_4 \right| = (1 - c_e) \sqrt{(m_\nu)^2_{min} + \delta m^2} \sqrt{1 - \sin^2 2\theta_{sun}} \tag{79}$$

where $\delta m^2 = \delta m^2_{atm}$ (for B_3) and $\delta m^2 = \delta m^2_{atm} + \delta m^2_{LSND}$ for B_4. As $0 \leq c_e \leq 0.05$

$$w_{\min} - \left(|U_{e3}|^2 m_3 + |U_{e4}|^2 m_4 \right) < c_e (m_\nu)_{min} - (1 - c_e) \sqrt{(m_\nu)^2_{min} + \delta m^2_{atm}} < 0, \tag{80}$$

and the first two regions in Eq. (73) are not present. In the B_4 scheme, as in the A_4, we do not know $|U_{e1}|^2$ and $|U_{e2}|^2$ separately.

For $w_{\max}$ only the bound can be found

$$c_e (m_\nu)_{min} < w_{\max} < c_e \sqrt{(m_\nu)^2_{min} + \delta m^2_{atm}}. \tag{81}$$

In this case the $|\langle m_\nu \rangle|$ satisfies

$$|\langle m_\nu \rangle|_{\min} \geq (1 - c_e) \sqrt{(m_\nu)_{min}^2 + \delta m^2} \sqrt{1 - \sin^2 2\theta_{sun}} - c_e m_{min} \equiv f\left[(m_\nu)_{min}\right] \tag{82}$$

where $m_{min} = (m_\nu)_{min}$ for B_3 and $m_{min} = \sqrt{(m_\nu)_{min}^2 + \delta m_{atm}^2}$ for B_4.

If condition Eq. (69) is satisfied $f\left[(m_\nu)_{min}\right]$ is an increasing function of $(m_\nu)_{min}$, if not, $f\left[(m_\nu)_{min}\right]$ decreases from

$$f\left[0\right] = (1 - c_e) \sqrt{\delta m^2} \sqrt{1 - \sin^2 2\theta_{sun}} - c_e m_{min}\left[(m_\nu)_{min} = 0\right] \text{ for } (m_\nu)_{min} = 0 \tag{83}$$

to

$$f\left[(m_\nu)_{min}\right] = 0 \text{ for } (m_\nu)_{min} = \left[\frac{\delta m^2 \left(1 - \sin^2 2\theta_{sun}\right) - \frac{c_e^2}{(1-c_e)^2} m_{min}\left[(m_\nu)_{min} = 0\right]}{\sin^2 2\theta_{sun} - \frac{(1-2c_e)}{(1-c_e)^2}} \right]^{1/2} . \tag{84}$$

All the above analytical considerations lead us to the following conclusions (for plots see [9]).

A) Present bound $|\langle m_\nu \rangle| < 0.2 \ eV.$ (see Fig. 3)

- If SMA MSW solution is the proper mechanism then:
  - B_4 scheme is excluded for Majorana neutrinos,
  - In schemes A_3, A_4 and B_3 Majorana neutrinos are accepted if $(m_\nu)_{\min} < 0.22 \ eV$. Above this mass all three schemes are open only for Dirac neutrinos.
- For LMA and LOW MSW solutions:
  - the A_3, A_4 and B_3 schemes accept Majorana neutrinos only for $(m_\nu)_{\min} < 1.5 \ eV$, an analogous limit in the B_4 scheme is $(m_\nu)_{\min} < 1.1 \ eV$.

B) If GENIUS I gives only a bound $|\langle m_\nu \rangle| < 0.02 \ eV :$ (see Fig. 3)

- For SMA MSW solution:
  - scheme B_3 is excluded for Majorana neutrinos,
  - in schemes A_3 and A_4 Majorana neutrinos are accepted only for small masses $(m_\nu)_{\min} < 0.04 \ eV$.
- For LMA and LOW solutions:
  - the B_4 scheme is excluded for Majorana neutrinos,
  - Majorana neutrinos can exist for $(m_\nu)_{\min} < 0.16 \ eV \ (A_3)$, $(m_\nu)_{\min} < 0.14 \ eV \ (B_3)$ and $(m_\nu)_{\min} < 0.22 eV \ (A_4)$.
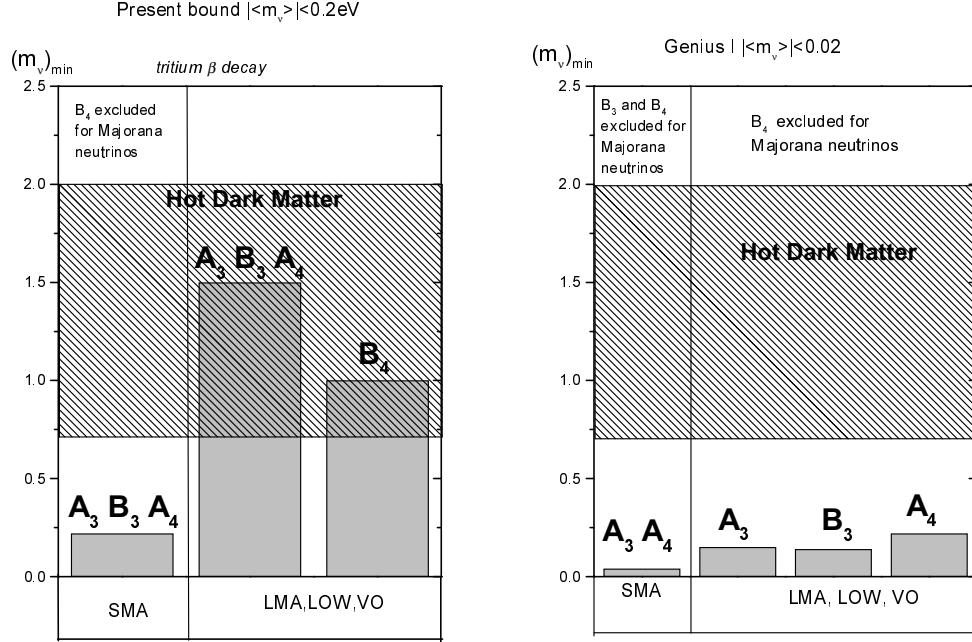
Fig. 3. Upper limits on the mass of the lightest of Majorana neutrinos derived from present (left) and GENIUS I (right) $(\beta\beta)_{0\nu}$ experimental bounds for different neutrino mass schemes and various solar neutrino oscillation solutions. The gray shaded area shows the allowed mass region for this neutrino. The HDM area applies in the three neutrino case only. We can see that Genius I with HDM solve the problem of neutrinos' nature in this case.

C) If finally GENIUS II does not find the $(\beta\beta)_{0\nu}$ decay (see Fig. 4):

- For SMA MSW solution:
  - Majorana neutrinos with $(m_\nu)_{\min} \leq 0.02 eV$ can exist only in the A_3 and A_4 schemes
- For LMA MSW and LOW solutions
  - the B_3 scheme is excluded for Majorana neutrinos,
  - Majorana neutrinos can exist only in A_3 and A_4 schemes with $(m_\nu)_{\min} < 0.05$ eV and $(m_\nu)_{\min} < 0.12$ eV, respectively.
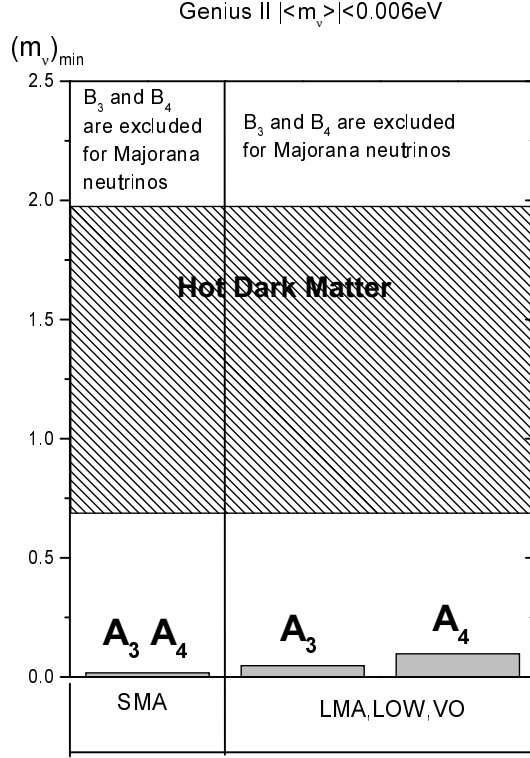
Fig. 4. Even more restricted area allowed for the mass of the lightest Majorana neutrino in the case of GENIUS II.

There are additional restrictions with assumption that neutrinos contribute to the dark matter content. Three neutrinos with almost degenerate masses $m_\nu \sim 0.7\ eV\ (2eV)$ must exist if $\sum m_\nu \approx 2\ eV\ (6\ \ eV)$. This means that already the present $(\beta\beta)_{0\nu}$ bound closes all schemes for three Majorana neutrinos if the SMA solution is the proper one. The GENIUS I bound will close schemes for three Majorana neutrinos. For schemes A_4 and B_4 with a sterile neutrino $(m_\nu)_{\min}$ must be very small if $\sum m_\nu \simeq 2\ eV$ and $(m_\nu)_{\min} \approx 1.0$ if $\sum m_\nu \approx 6\ eV$. Then the only scheme with one sterile neutrino is accepted if the sum of all Majorana neutrinos is approximately 2 eV. If $\sum m_\nu \sim 6\ eV$ and GENIUS I will give negative results only 3 or 4 Dirac neutrinos can constitute the HDM. In such case there is a problem how to

explain the number of neutrino degrees of freedom from the abundance of the ^{4}He and D/N. The present highest bound is $N_\nu < 5.3$ [42].

5. Conclusions.

We have entered an exciting era in neutrino physics. Mixing in the lepton sector seems to be established. An obvious consequence of this fact is the nonconservation of lepton family number L_α. Breaking of the L_α is very weak. It is seen only in neutrino oscillation and in no other terrestrial laboratory experiments. The problem of the conservation or violation of the total lepton number L, which is connected with the Dirac or Majorana neutrino nature, is not solved up to now. Approximate conservation of L_α and L follows from i) smallness of neutrino masses, ii)ultrarelativistic character of produced neutrinos, iii) unitarity (exact or approximate) of the mixing matrix, iv) left-handed nature of the neutrino interaction.

For Majorana neutrinos this approximate L_α and L conservation can be proved even though lepton numbers are not defined for neutral particles.

The Majorana neutrino mass matrix elements $m_{\alpha\beta}$ ($\alpha, \beta = e, \mu, \tau$) are bounded by various experiments. Such bounds are usually in the MeV-GeV range. Only one element m_{ee} is limited with good enough precision to play a role in the reconstruction of the mixing in the lepton sector. The m_{ee} element is measured in double β decay of various nuclei. Up to now this decay has not been observed. The contrary would establish the Majorana nature of neutrinos. However the combination of various informations about masses and mixing matrix elements from i) oscillation experiments, (ii) tritium β decay and (iii) cosmology together with $(\beta\beta)_{0\nu}$ is able to discriminate between the accepted neutrino mass spectra allowed for Majorana or only for Dirac neutrinos. The data are not precise enough to make conclusive statements. The bound on $\langle m_\nu \rangle$depends strongly on the determination of nuclear matrix elements. Our estimation was made with 95% CL. At 3σ which corresponds to 99% CL, a value of one for $\sin^2 2\theta_{sun}$ is accepted and we cannot make any discrimination between the two natures.

Our estimation is interesting also for those who strongly believe that neutrinos are Majorana particles. We found the corner of the mass schemes where such neutral particles are still allowed. With the present experimental precision the room for the Majorana neutrino is bounded but still large. If next $(\beta\beta)_{0\nu}$ experiments give negative results the Majorana neutrino corner will become smaller and smaller. More precise informations about i) existence of sterile neutrino, ii) which solution of the solar neutrino anomaly is accepted and iii) knowledge of oscillation parameters with smaller error are urgently needed. We hope that future (already working and planned) experiments will provide us with these informations and together with the

neutrino mass scheme, the neutrino character will be established.

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